Universität zu Köln
Markets for Risk Management

Problem Set #1 Solutions
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Problem #1: Demand for Insurance (33 points)

Suppose you wish to insure an asset valued at €100. Only two states of the world can occur in the future, FIRE and NO FIRE, with probabilities .25 and .75 respectively. In the FIRE state, the asset is completely destroyed. Your initial wealth (including this asset) is €120, and your utility $U(W) = \ln W$.

1. (11 points) Suppose an insurer offers to fully insure your fire risk for a premium of €25. Should you purchase this insurance policy? Why or why not?

   Solution: Since $E(L) = .25(€100) = €25$, this represents an actuarially fair price. According to the Bernoulli principle, risk averters will find it optimal to fully insure when insurance is actuarially fair. Therefore I would purchase this policy.

   The fact that it is optimal to purchase this policy can be confirmed by calculating the expected utility of being fully insured and comparing this with the expected utility of being self-insured. By purchasing insurance for €25, this means that I have a choice between certain wealth of €95 (full insurance case) and a lottery with an expected value of €95 (self-insurance case):

   Full insurance: $E(U(W)) = .25(\ln(95)) + .75(\ln(95)) = \ln(95) = 4.5539$.

   Self-insurance: $E(U(W)) = .25(\ln(20)) + .75(\ln(120)) = 4.3396$.

2. (11 points) If the premium for full coverage is €35, should you fully insure? Why or why not?

   Solution: By purchasing insurance for €35, this means that I have a choice between certain wealth of €85 (full insurance case) and a lottery with an expected value of €95 (self-insurance case):

   Full insurance: $E(U(W)) = .25(\ln(85)) + .75(\ln(85)) = \ln(85) = 4.4427$.

   Since full insurance has higher expected utility than self-insurance, I would prefer to fully insure.

3. (11 points) What is the maximum premium you are willing to pay to fully insure this risk? Explain how you determined the answer to this question.

   Solution: The maximum premium is equal to the actuarially fair premium plus the risk premium, which is calculated as the difference between expected wealth minus the certainty equivalent of wealth under the self-insurance option. Since I have $U(W) = \ln(W)$ and $E(U(W)) = 4.3396$, my certainty equivalent of wealth is $e^{4.3396} = €76.67$, my risk premium is E(W) – W_{CE}= €95 - €76.67 = €18.33, and the maximum premium I am willing to pay to fully insure this risk is €25 + €18.33 = €43.33.

Problem #2: Moral Hazard (33 points)

The sole owner and manager of EMH, Inc. is a fellow by the name of Fama. Fama is risk averse, with utility $U(W) = W^{0.5}$. The EMH factory facility is worth €3 million, and the main business risk facing EMH, Inc. is a factory fire. If the EMH factory burns down, the company will suffer a complete loss of €3 million. Fama also has €1 million in the bank that is *not* at risk. His initial wealth (W) therefore consists of his €3 million factory plus his €1 million bank account.

The probability of a factory fire depends in part upon whether Fama runs a fire prevention training program at the EMH factory. It costs €100,000 to run such a program. If he does not run the program, probability of a factory fire is 5%. However, if he runs the program, this would reduce the probability of a factory fire down to only 1%.

In parts B and C below, Ace Insurance Company is a risk-neutral insurer who cannot observe whether Fama conducts the fire prevention program.

1. (11 points) If Fama has no insurance, will he run the fire prevention program? Why or why not?

   SOLUTION: Fama's expected utility if he runs the fire prevention program is

   $\boldsymbol{EU}(\Pi)_{RunProgram} = 0.99*(4{,}000{,}000 - 100{,}000)^{0.5} + 0.01*(1{,}000{,}000 - 100{,}000)^{0.5}$

   $= 0.99*(3{,}900{,}000)^{0.5} + 0.01(900{,}000)^{0.5}$

   $= 0.99*1{,}974.842 + 0.01*948.683$

   $= 1{,}955.093 + 9.487 = 1{,}964.580$

   Fama's expected utility if he doesn't run the fire prevention program is

   $\boldsymbol{EU}(\Pi)_{NotRunProgram} = 0.95*(4{,}000{,}000)^{0.5} + 0.05*(1{,}000{,}000)^{0.5}$

   $= 0.95*2.000 + 0.05*1{,}000$

   $= 1{,}900 + 50 = 1{,}950$

   Since $1{,}964.580 > 1{,}950$, Fama runs the fire prevention program.

2. (11 points) What is the maximum price that Fama would be willing to pay for full insurance coverage?

   SOLUTION: From a utility standpoint, Fama's next best risk management alternative to purchasing a full coverage insurance policy is to go without insurance and mitigate the fire risk by running a fire prevention training program at the EMH factory. Thus, Fama will be indifferent about purchasing a full coverage policy if Ace Insurance Company charges him a premium which will reduce the utility value of the certainty equivalent of wealth down to the utility value associated with remaining uninsured and mitigating by running a fire prevention training program at the

EMH factory, which is 1,964.58. The certainty equivalent of 1,964.58 is $(1{,}964.58)^2$ = €3,859,575; thus, EMH, Inc. purchased a full coverage insurance certain profit of €3,859,575 for sure, then Fama's utility would have utility of $3{,}859{,}575^{0.5} = 1{,}964.58$ (the same utility for sure as the expected utility if he didn't run the fire prevention program). Ace Insurance Company can guarantee that EMH, Inc. will have profit of €3,859,575 for sure by charging €140,425 for full coverage insurance. Thus, the maximum price that Fama would be willing to pay for full coverage insurance is

4,000,000 – 3,859,575 = €140,425.

3. (11 points) What is Ace Insurance Company's expected profit or loss if it offers EMH, Inc. the full insurance policy for the maximum price in part (B)?

   SOLUTION: Fama will *not* run the fire prevention program because EMH, Inc. can receive full damages from Ace without having to spend the €100,000 for fire prevention. If Fama does not run the fire prevention program, Ace's expected payout to EMH, Inc. is 0.95*0 + 0.05*3,000,000 = €150,000. If Ace sells the policy for 140,425, Ace's expected loss is 140,425 – 150,000 = -€9,575.

Problem #3: Adverse Selection (34 points)

You are an insurer. All of your clients are arbitrarily risk averse; in the absence of insurance, each one of them starts out with €150 in initial wealth and loses €100 if they are involved in an accident. The insurance policies you can offer them are also shown, as well as the insured wealth under each policy:

	premium	Indemnity (payment by the insurer to the client)	wealth in non-loss state	wealth in loss state
uninsured wealth			€150	€50
policy A	€12	€44	€138	€94
policy B	€26	€100	€124	€124
policy C	€76	€100	€74	€74

Half of your clients have a probability of loss equal to .75, the others have probability of loss equal to .25. They know their risks but you don't. You do know that the high-risk clients rank the contracts, best to worse, in this order:

- High risk clients' insurance preferences: policy B is preferred to policy C, policy C is preferred to policy A, and policy A is preferred to not being insured.

The low-risk clients prefer the contracts in this order:

- Low risk clients' insurance preferences: policy B is preferred to policy A, policy A is preferred to not being insured, and not being insured is preferred to policy C.

Which policies should you offer if you are interested in maximizing the expected value of profit? You choices are to offer 1) all three policies, 2) policies A and B, 3) policies A and C, 4) policies B and C, 5) just policy A, 6) just policy B, 7) just policy C, and 8)

no policies. Explain carefully by showing that your choice produces a higher expected value of profit than any of the other possible choices that you can make.

Solution: Calculate the expected profit on each contract for each group, filling out two additional columns of the table:

	premium	Indemnity (payment by the insurer to the client)	wealth in non-loss state	wealth in loss state	expected profit	expected profit
uninsured wealth			€150	€50	high risk	low risk
policy A	€12	€44	€138	€94	€12 -.75 * €44 = -€21	€12 -.25 * €44 = €1
policy B	€26	€100	€124	€124	€26 -.75 *€100 = -€49	€26 -.25 * €100 = €1
policy C	€76	€100	€74	€74	€76 -.75 * €100 = €1	€76 -.25 *€100 =€51

If you offer all three contracts, the high risk and low-risk clients both take contract B, and you lose money. Therefore you can never offer B. Try A or C or both. If you offer both A and C the high-risk types will take C and that contract will have an expected profit of €1. The low risk types will take A and that contract will also have an expected profit of €1. Offering only A won't work because it will attract a mix of high-risk and low-risk clients and will therefore lose money. Offering only C will make less profit than A and C together. Therefore, the way to maximize the expected value of profit is to offer policies A and C.